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Statistical Energy Analysis of structural vibrations: A review

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Abstract— The paper presents futuristic review on statistical energy analysis of structural vibrations. Statistical Energy Analysis is a structural-acoustic method that is widely used for response prediction at high frequencies. Statistical energy analysis arose during the 1960´s in the aerospace industry to predict the vibrational behaviour, while designing spacecrafts. Statistical energy analysis has been used as an estimation technique for mid and high frequency vibration and sound transmission. Statistical energy analysis has been proven successful and has a wide application ranging from buildings to spacecrafts. Statistical energy analysis is a method for predicting the transmission of sound (energy levels) and vibration through complex structural acoustic systems.

Index Terms— vibro-acoustic, coupling loss factors, coupling strength, modal overlap, high frequency, sound transmission, vibration energy level.

1 Introduction

Statistical Energy (SEA) Analysis is one of the widely used energy methods, developed in the early 1960s to predict the vibration response of structures at high frequencies, pioneered by Lyon [1]. The theoretical technique for the SEA applied to structural vibration was later given by Woodhouse [2]. SEA involves predicting the vibration response of a complex structure by dividing it into a number of subsystems (Fig. 1.), and is characterized by mean energy per mode and the change in energy level between subsystems, that is characterized by loss factor comprising of internal loss and coupling loss factor. Internal loss factor corresponds to damping in the subsystem itself and coupling loss factor corresponds to the energy dissipation during flow across the subsystems.

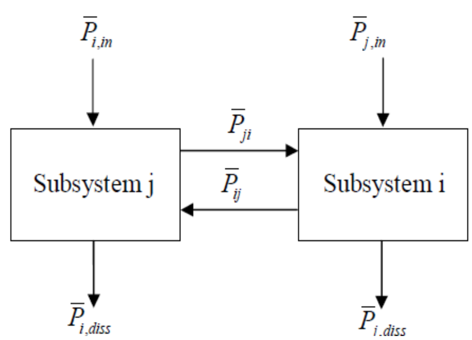

Fig. 1. Energy flow across two Subsystems

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(This information is optional; change it according to your need.)

The power balance equation for n systems is given by,

$$\omega\begin{bmatrix} \left(\eta_1+\sum_{i\neq 1}^{N}\eta_{1i}\right)n_1 & -\eta_{12}n_1 & \cdots & -\eta_{1N}n_1 \\ -\eta_{21}n_2 & \left(\eta_2+\sum_{i\neq 2}^{N}\eta_{2i}\right)n_2 & \cdots & -\eta_{2N}n_2 \\ \vdots & \vdots & \ddots & \vdots \\ -\eta_{N1}n_N & \cdots & \cdots & \left(\eta_N+\sum_{i\neq N}^{N-1}\eta_{Ni}\right)n_N \end{bmatrix}\times\begin{bmatrix} \frac{\langle\bar{E}_1\rangle}{n_1} \\ \frac{\langle\bar{E}_2\rangle}{n_2} \\ \vdots \\ \frac{\langle\bar{E}_N\rangle}{n_N} \end{bmatrix}=\begin{bmatrix} \bar{P}_{i,1} \\ \bar{P}_{i,2} \\ \vdots \\ \bar{P}_{i,N} \end{bmatrix} \quad (1)$$

The power balance equation is given by above equation, where, Pi,n is the power injected in subsystem i, <E> is the frequency averaged energy in subsystem i. The bar above symbol indicates the frequency averaging, < >, indicates spatial averaging, ηi, is the internal damping loss factor in subsystem i, ηij, is the coupling loss factor from subsystem i to subsystem j. The coupling loss factor ηij is independent of power input and dependent on the internal loss will define the vibration energy level transferred to subsystems. From SEA point of view [3-6] the coupling should be linear, conservative and weak. The input power on each subsystem is random and statistically independent of the forcing on other subsystems. For justifying for average modal energies, the modes are assumed to be equally excited in each subsystem. The exchange of energy with the coupled subsystems happens to be with large number of modes present in the frequency band of interest, ignoring the non-resonant modes. It considers all mode shapes equally probable, so that the effect of modes at the coupling and spatial modal coherence is ignored.

In vibro-acoustics engineering, the audible frequency range is divided into low, medium and high frequency region. There is no specific frequency band to distinguish the low, medium and high frequency region. Generally the magnitude of resonance is high in low frequency region, also the effects of boundary conditions, damping and geometrical variability are clearly observed. Whereas high frequency region due to modal overlapping the response curve will be smooth. There is no clear method to distinguish the low, medium and high frequency regions, but for structural problems modal overlap factor can be used as an approximation method to indicate high frequency threshold as given by Rabbiolo [7]. The author specifies that the modal overlap factor for beams should be 1, for plate like structures should be 2.5 and 3 for acoustic elements.

The vibraation analysis of complex plate like structures subjected to high frequency vibrations becomes unsuitable Finite Element (FE) method, since the problem size becomes enormous. In contrast, SEA works well at high frequencies, but is less reliable in the medium-frequency range where the coupling between subsystems is strong and the modal density is low. The accuracy of SEA hinges on the Coupling Loss Factors (CLF), which is traditionally determined from classical wave approach, experimental, FEM or hybrid of these, involving a number of limiting assumptions.

Coupling losses in structural systems consisting of beams and plates are often determined using wave theory in case of classical SEA. The coupling loss factors obtained from wave theory transmission were estimated that predicted from computational [8] and experimental [9] when the two coupled elements (beams and plates) have modal overlap factor is greater than or equal to unity. Though it is not exact, for making the calculations easier the modal overlap factor can be considered as a practical indicator. Mace [10] showed that coupling strength depends more on a coupling strength than that of modal overlap factor for rectangular plates. Fahy [11] in his work on plates suggested an extra condition other than modal overlap factor that, there should be at least five modes in the frequency band. Hence, for classical SEA wave theory the modal overlap factor should be greater than unity and mode counts should be greater than five in the frequency band for plate systems.

J Woodhouse [2] in his paper gave an insight on the terms of reference in SEA model and procedure for determining the SEA parameters experimentally. He also stated that, studying the behaviour of structures subjected to mid or high frequency range SEA is more feasible and useful over deterministic analysis. This is because at high frequencies modal density will be high and more number of modes has to be considered. These modes are more sensitive to the small structural variations and hence the numerical accuracy decreases. For complicated models it becomes extremely difficult to draw the inference as the volume of output will be high.

The approach suggested by Woodhouse [2] in determining the SEA parameters, led to the problem in matrix fitting couldn't be optimized. A numerical procedure by using two simple algorithms was developed for optimizing the results were outlined by Hodges [12]. The degree of accuracy of predicting the SEA parameters and consistency of SEA model lies with matrix fitting. The SEA model and inverse of the matrix found by Lagrange multiplier algorithm found to best suited.

Woodhouse [13] investigated damping effects on energy sharing in coupled systems. The SEA parameters were computed using matrix inversion method. As per the classical theory the damping reduces the vibration energy in the system, and reduces the statistical parameters in the SEA. He inferred that the coupling loss, a SEA parameter is proportionally dependent on the damping only at lower values and as the damping values increases, the coupling loss becomes independent.

2 Analytical Modelling

SEA is one of the widely used energy methods, developed in the early 1960s to predict the vibration response of structures at high frequencies, pioneered by Lyon [1]. The theoretical technique for the SEA applied to structural vibration was later given by Woodhouse [2, 12]. In SEA the complex structure subjected to high frequency vibration is divided into a number of subsystems, and is characterized by mean energy per mode. The change in energy level between subsystems are defined by the loss factors comprising of internal loss and coupling loss factor. Since the wavelength is shorter at high frequency, prediction of vibration levels analytically is difficult. The discretised length of each element should be at least 1/8th of the wavelength for numerical method, which increases the computational efforts at high frequencies. Hence most of the research was devoted in developing the alternatives for predicting the vibration response at high frequencies. Langely and Wester [14-15] stated that though SEA was used widely for studying the response of different structures, the validity and the reliability of the results was still a question since it used to yield poor results for some structures.

Fahy [16] studied the SEA parameters in the stiffened panels widely used in aerospace and marine vehicles and the modelling of these stiffened panels in SEA model was addressed. The stiffened panels were separately modelled and it represented the coupling element. These coupling elements associated with the coupling loss factors were computed by considering the wave transmission across stiffener studied by Langley [17]. He concluded that for determining SEA parameters in stiffened panels Wave Intensity Analysis (WIA) offered better results over standard SEA.

Shankar [18] stressed on consideration of the indirect coupling losses which was neglected in conventional SEA. Based on the geometrical construction of the system, the total coupling loss factors can comprises of direct and indirect factors. The couplings between the subsystems which are not connected physically refer to indirect, while the subsystems which are connected physically refer to direct coupling loss factors. The generalized form of receptance theory and Green functions applicable to many subsystems coupled by an arbitrary number of springs was also investigated by the author. The study of energy flow between two weakly coupled beams using Green function was stated by Davies [19, 20]. Later it was extended to derive power flows between coupled axially vibrating rods by Remington and Manning [21]. It was Langley [22] who showed that modal summations over uncoupled system modes as Green functions. The expression for energy flow in terms of the uncoupled of coupled axially vibrating rods was given by Keane and Price [23]. Later Keane [24] came up with a consistent method applicable for many subsystems coupled by an arbitrary number of springs.

Gaul [25] showed with an example that, the unavoidable source of hysteresis is due to the theoretical assumption in SEA that the joints between the subsystems are weak and conservative. But these joints between the subsystems, dissipate more energy than structural damping itself, and are no longer weak and conservative experimentally.

The energy flow between two non-conservatively coupled oscillators was also studied by Fahy and Yao [26], and concluded that the energy flow depends not only on the absolute energy values but also on the difference between these energies. He also showed that for controlling the energy of the indirectly driven subsystem the coupling damping should have the same order as that of internal damping. The modified SEA energy balance equation was given by Sun [27] who carried the work on non-conservatively coupled continuous structures. His work showed that coupling damping increases the effective internal loss factors of the substructures. Chen [28] also carried his work on non-conservatively coupled oscillators, having different values of natural frequencies among the oscillators. With the different values of natural frequencies among the two non-conservatively coupled oscillators the coupling damping increases the energy flow from driven to undriven oscillators. For non-conservatively coupled oscillators, the coupling damping also increases the energy flow from driven to the undriven oscillators, when the natural frequencies of the two oscillators are different was depicted in the work of Chen and Soong [28]

Beshara [29] extended the work of Keane [24] in determining the energy flow for many subsystems having non-conservative couplings. He made the first break through in studying the energy flow for multi-modal subsystems having non-conservative couplings by modeling the joints of the subsystems using spring and damper. He used energy flow receptances approach to determine the response of a complex built-up system in terms of the responses of each individual subsystem. The Deterministic expressions in the frequency domain for both point and "rain-on-the-roof" driving were given to determine the energy levels and energy dissipations in various parts of overall system. Through numerical examples, the effects of change in coupling damping on various power receptances were studied for "rain-on-the-roof forcing. The damper was provided with an intention of dissipating some amount of energy between the joints. He suggested a SEA model taking into account the coupling damping loss factor in the energy balance equation.

Further, Beshara [29] in his work also made a significant note on power injection method. The author stressed on the effect of damping in the joints on the energy flow in the subsystems, which was ignored in traditional SEA. A new model was proposed by introducing the coupling damping loss factors into the main SEA energy balance equation to consider the energy dissipation in the joints especially in the case of non-conservative couplings. It was understood that the internal loss factors largely depends on the type of materials subsystems is made of. Once the sum of these internal loss factors and the coupling damping loss factors are determined, then one can evaluate the significance of the coupling damping loss factors and the errors resulting from ignoring damping in the joints while manoeuvring with traditional SEA equations. The coupling loss factors and the total subsystem internal loss factor The power injection method is used to obtain the coupling loss factors and the total subsystem internal loss factors.

Shankar [30] formulated a rigidly coupled beam network using receptance theory for estimating the energy flow in the coupled beam and also predicting the behaviour of the global structure of the uncoupled elements. In his study the green function matrices were used to derive the displacement functions for the beam-end, resulting by the contribution of unknown boundary coupling forces and external driving forces. The energy flow through the various substructures were obtained after solving for unknown boundary coupling forces as result of number of complex simultaneous linear equations imposed by applying suitable boundary conditions at the beam-ends.

Beshara [31] carried out an investigation on two thin rectangular plates using receptance approach, to know the transmission of energy flow through a compliant and dissipative joint. Vaicaitis [32] has discussed different types of such joints found in many real structures used in aircraft or buildings. Beshara in his work, applied point force and "rain-on-the-roof" as driving forces for the considered thin rectangular plates and established the exact formulae for the spectral density of the energy flow through the joint, the power input by the external excitation, the energy dissipated at the joint and vibrational energies of the two plates.The main aim of his paper was to show that, when the couplings are weak the joint damping is most effective and for any combination of joints and the value of damping, the input power is dissipated in the joints, minimizing the energy levels in the plates that are not directly excited with the applied forces.

Fahy [33] discussed about coupling strength (Cs) and had shown that when its value is beyond 0.07 would represent the weak coupling, provided the resonance frequencies of the uncoupled subsystems are sufficiently proximate. Even in case of weak coupling, the coupling strength depends on the proximity of the frequency between the modal frequencies of each uncoupled subsystems and can give values less than 0.07.

James [34] proposed modal interaction (Mp) as an indicator to overcome the difficulty in exegesis on small values of coupling strength. Modal interaction can be defined as the state to have Cs greater than 0.07 for all cases of weak coupling when the resonance frequencies of the uncoupled subsystems are sufficiently proximate. James in his work considered two coupled rods, two coupled beams, two rectangular coupled plates for numerical simulations and two coupled plates, two rooms coupled by an aperture for experimentation to determine the modal interaction using the estimated values of modal densities and internal loss factors of the subsystems. It was shown that the coupling strength was reliable in a SEA sense when the modal interaction value was greater than unity. On the other hand, when the coupling strength is less than 0.07 and modal interaction is also less than unity, then distinguishing the strong and weak coupling was impossible. It was shown that modal interaction and coupling strength was interdependent.

Ji [35] introduced coupling co-efficient parameter along with coupling strength parameter which also accounts for statistics of coupling stiffness. Expressions for ensemble means and variances of the subsystem energies were derived considering two rectangular plates coupled by spring coupling. Appropriate estimation of coupling strength parameter was made by taking various approximations and assumptions. The theory proposed by Ji showed that the variance of the input power depends on the variance of the number of modes of the excited subsystems and their shapes. Also the variance of the excited subsystem depends on the variance of the input power. Furthermore, the variance of the intermodal coupling co-efficients depends on the variaces of the number of in-band modes of both subsystems and thier shapes. Also the variance of the undriven subsystem depends on the variance of the intermodal coupling co-efficients.

3 Numerical Modelling

Shankar [18] used structural dynamics finite element package, to compute the energy flow in the coupled systems, by extracting the displacement and velocity responses from the software package. He made his remarks that, though the commercial software packages were designed for low frequency modal analysis, one can derive the power and energy flow in the coupled systems from the displacement and velocity responses by performing certain post-processing operations. He studied the behaviour of hinged-hinged beams, point coupled by springs with weak to strong coupling from the point of vibrational energies and input power transferred through the coupling. The study was made by considering two configurations, one by considering only the transverse Euler-Bernoulli modes by arranging the beams parallel to each other, and the other by considering both axial and transverse modes by arranging the beams perpendicular to each other. The FEA analysis of the beam subsystem was modelled by using IDEAS-VI linear beam having 440 DOF for the first configuration and 658 for second configuration.

Shankar [18] concluded that in case of very strong and very weak coupling the coupling power evaluated by considering the difference in displacement responses in FEA approach are not appropriate because of numerical limitations. The indirect coupling loss factors play a vital role in computing the coupling loss factors for the system having more than two subsystems, especially if not all the subsystems are driven.

Shankar [36] presented the study of vibrational energy flow by considering the structure made up of substructures modelled by using finite element analysis based on receptance theory. Each substructure was analysed for its eigen values and eigen vectors using free-free interface conditions. The iterative eigen value solvers of IDEAS-VI [37] package was used for finite element analysis, uses Simultaneous Vector Iteration (SVI) algorithm. Though the Householder-QL and Jacobi methods also reduces the system matrix to diagonal form through matrix rotations, the former being faster as it performs all the operations in core, making it suitable for small models. While the Householder-QL writes the intermediate data to disk, making it suitable for lager models.

Hopkins [38] determined the coupling loss factors for statistical energy analysis using finite element methods, experimental statistical energy analysis (ESEA) and Monte Carlo methods. As stated earlier in the range of medium frequency where the coupling strength is high and have low modal density the SEA is not reliable, for which Hopkins proposed ESEA for the systems with low modal density and low modal overlap. The conventional SEA was not appropriate for single deterministic approach following the failure of matrix inversion. This was not encountered in majority of the ESEA ensemble. It was concluded that ESEA ensemble was advantageous in determining coupling parameters over single deterministic analysis since in the later method, there will be lot of variation in coupling parameters with small variations in the physical properties of the structure with low modal overlap and low modal density. The FEM analysis was carried out using Ansys-5.5 software considering plate like structure, meshed with Shell-63 element having the element length less than 1/6th of bending wave length. The unconstrained nodes of the source plate were subjected to rain-on-the-roof excitation with unit magnitude and random phase. The damping ratio which is constant is treated purely as internal loss factor in FEM calculation. But, to determine the total loss factor in the coupled structure sum of internal loss factor and coupling loss factor are to be considered [39], which was quiet different from what Simmons [40] where it was dominated by internal loss factor for thin plates.

Guyader [41] developed a new technique called Statistical modal Energy distribution Analysis (SmEdA). It was observed that in case of low modal overlap i.e., even when there is no equi-partition of energy in the subsystems, the coupling factor results obtained by SmEdA were more precise than that of standard SEA. A rectangular plate simply supported to a cavity with point force excitation was used along with fuzzy number theory to determine average ensemble energies in the subsystems.

4 Experimental SEA

The Statistical energy analysis proposed by Lyon [1] to analyse the structural dynamics in the range of high frequencies also highlighted the main problems in measuring the energies which would result in obtaining negative coupling loss factors due to ill-condition matrices. Later Woodhouse [2] stressed on ESEA that the later can be used to verify whether SEA is suitable to assess the effect of any change in the system under study and also allows in determining SEA parameters of complex junctions which is often inaccurate or not possible theoretically.

Hopkins [38] determined the coupling loss factors for statistical energy analysis using experimental statistical energy analysis by modelling isolated L- and T-junction of rectangular plates. The loss factor is determined by inversing the energy matrix obtained by experimentally finding the subsystem

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energies and power inputs. The subsystem energies are measured experimentally in all subsystems by sequentially injecting the power in each subsystem. The systems should be equipartitioned suitably so as to have negligible error in the energy computed and to have weak coupling for ESEA, the energy matrix should be well-conditioned.

The general ESEA matrix [38] is determined from the general SEA matrix and is shown in equation below, where Eij is the energy of subsystem i with power input into subsystem j, ηij is the CLF from subsystem i to subsystem j, ηii is the internal loss factor (ILF) of subsystem i, and Pin,1 is the power input to subsystem i.

$$\begin{bmatrix} \sum_{n=1}^{N}\eta_{1n} & -\eta_{12} & \cdots & -\eta_{N1} \\ -\eta_{12} & \sum_{n=1}^{N}\eta_{2n} & \cdots & -\eta_{N2} \\ \vdots & \vdots & \ddots & \vdots \\ -\eta_{1N} & \cdots & \cdots & \sum_{n=1}^{N}\eta_{Nn} \end{bmatrix} \times \begin{bmatrix} E_{11} & E_{12} & \cdots & E_{1N} \\ E_{21} & E_{22} & \cdots & E_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ E_{N1} & \cdots & \cdots & E_{NN} \end{bmatrix} = \begin{bmatrix} P_{in,1}/\omega & 0 & \cdots & . \\ 0 & P_{in,2}/\omega & \cdots & . \\ \vdots & \vdots & \ddots & \vdots \\ 0 & \cdots & \cdots & P_{in,N}/\omega \end{bmatrix} \quad (2)$$

Fahy [33] conducted experiments with two coupled plates and two coupled rooms and suggested the technique for the assessment of coupling loss factors between subsystems. To conduct the experimental SEA analysis the system has to be divided into subsystems optimally such that the subsystems are weakly coupled. If the subsystems have strong coupling then the experimental values obtained by power injection method will have errors since both the subsystems have similar modal energies. The conditions for weak coupling in the subsystems as proposed [42-45] earlier, only few had experimental significance. It was shown that the coupling strength beyond 0.07 would represent the weak coupling provided the resonance frequencies of the uncoupled subsystems are sufficiently proximate. The technique for the assessment of coupling loss factor was made on the dimensionless parameter called "Coupling Strength". The experiment was carried out considering two plates connected by straps. The number of straps connecting the plates decides the coupling strength and varying the number of straps used to connect the plates varies the coupling strength directly. It was also further extended for rooms with aperture and studied about the coupling strength.

Fahy [46], to overcome the difficulty in measuring the input power suggested an alternate method to conventional SEA and called it as "power transfer coefficient". He also argued that the coupling loss coefficients determined by conventional SEA is not appropriate coefficient as it depends upon physical quantities which do not have any direct impact on relating the vibration levels between the subsystems. The alternate method suggested was based on calculating the SEA factor, "Strength-of-coupling" which was given by B R Mace and S Finnveden, [47-48].

Experimental SEA helps in analysing the plates and beams like structure with different configurations subjected to bending wave transmission. Some plate junctions where wave transmission happens may include in-plane waves along with bending waves. Experimental SEA doesn't support in-plane waves instead only bending waves. The wave transmissions at the junctions are completely neglected in experimental SEA. Hopkins [49] in his book titled "Sound Insulation" has given the details of existence of transverse shear and quasi-longitudinal wave subsystems which have low mode counts and low modal overlap in the plates subjected to bending wave motion above 500 Hz. It is difficult to accurately measure the in-plane wave motion in presence of bending vibration because of which only the bending waves were quantified to study SEA parameters between coupled systems was first sighted by Cimerman [50].

In practice, some type of plate junctions introduces significant wave conversion between bending and in-plane waves. No alternative technique for taking wave conversion effect into account was proposed. Hopkins [51] worked on this limitation of SEA that supports only bending waves. He proposed an improvised technique to account for the errors in the internal loss factor, matrix condition numbers and failure to satisfy the consistency relationships which were inherent in the experimental SEA. T-junction model was used in his work and comparisons were made between SEA and FEM techniques. Ansys software with shell 63 elements having one-sixth of the bending wavelength was used for FEM calculations. Hopkins in his work discussed about the effect of wave conversion on internal loss factor, matrix condition number and consistency relationship in experimental statistical energy analysis.

5 Other SEA

It is very important to know the structure-borne sound transmission in fields of noise control including aerospace engineering, ship, automobile and building. The structural dynamics for some of these structures can be carried out either by using deterministic or a statistical approach. These structures with low modal density and low modal overlap, the prediction of structure-borne sound transmission becomes difficult. Langley [52] proposed a technique that has the features of both deterministic and statistical approaches called the fuzzy structure theory. Single deterministic analysis will rarely predict the large fluctuations in the same frequency bands as in measured response because of uncertainty in material properties and dimensions in low and mid frequency range, which makes the response statistics more advantageous in the mentioned frequency range.

The coupling loss factor which is one of the important parameter in SEA determines the power flow between subsystems can be calculated theoretically for conservative couplings, and for non-conservative couplings can be determined through experiments [26-28]. While the internal loss factor which also computes for total loss factor has to be obtained experimentally. Determining these factors by matrix fitting method or numerical methods had few drawbacks for which Manik [53] suggested Power Injection Method (PIM). Some of the drawbacks include, the coupling loss factors turns out to be negative due to the use of ill conditioned matrix; to always have weak coupling which limits the possibility to use SEA for structures with strong coupling; and inconsistent coupling loss factors obtained due to obsolete the power injected data in few methods. The power injection method could overcome these drawbacks so as to obtain the coupling loss factors accurately irrespective of the strength of coupling used. The accuracy of coupling loss factors which would depend on the input power accuracy was determined using an impedance head and shaker or instrumented hammer and accelerometer for vibration measurement and sound intensity probe for sound power measurement.

Abdullah [54] introduced a novel scheme Discrete Singular Convolution (DSC) and Mode Superposition (MS) approach for studying discrete high frequency response of structures subjected to time-harmonic point forces. Mode superposition technique is mathematically derived from separation of variables. It assumes a solution considering all system modes that discretely contribute to local displacement response. By applying the boundary conditions the numerical scheme of DSC-MS is completed. A simply supported beam and thin plate were considered for the analysis and the results were compared against analytical method and found DSC-MS approach was much reliable than the latter.

Shorter [55] proposed a spectral finite element method to determine the dispersion properties of the first few wave types for the given composite laminate. The proposed technique was robust and numerically efficient alternative applicable for composite laminated whose material properties vary throughout the thickness of the section. The analysis was proposed to determine the wave propagation and damping in the viscoelastic laminates for one dimensional finite element mesh.

Sebastian [56] in his work applied the statistical energy analysis to orthotropic materials considering the curved laminates and sandwich composite panels. Discrete lamina description was used to represent the physical behaviour of the composite lamina. Each lamina was represented by membrane, bending, transversal shearing and rotational inertia behaviours. The results were compared with numerical and experimental tests and found to satisfactory for both sandwich panels and composite laminates. Further parametric study showed the variation of transmission loss with the change in arrangements of orthotropic plies and ply thickness.

Abdullah [57] introduced modal based approach, an approach derived from classical wave approach on symmetrically laminated right angled composite plates. Two symmetrically laminated composite plates were considered for numerical analysis, with 4 plies on plate 1 and 6 plies on plate 2. Two eight layered rectangular plates with zero degree orientation specially-orthotropic composite structure was considered for power injection method to compute the SEA parameters. The results obtained by model based approach were compared with the analytical approach for infinite orthotropic plates and also tested using power injection method. He concluded that the modal approach was not only the efficient tool to determine the SEA parameters in high frequency range but also in the medium frequency range applied for composite structures.

The classical SEA, based on its basic assumptions was restricted to steady-state vibration analysis. Lai [58, 59] established the energy balance relationship for structures subjected to transient excitations. These energy balance relations of transient excitations are similar to power flow balance relationship of steady-state vibration. Therefore the energy balance relation is used to describe the characteristics of transient SEA.

Pinnington [60] carried a numerical parameter study on two coupled beams subjected to impulse excitation. He computed peak velocity, peak energy and compared with modal overlap which was used as independent variable. The response was determined using exact wave solution and compared with approximate solutions a first wave prediction and a transient statistical energy solution. Two rigidly coupled beams having slight mismatch in the lengths, one with 10m and another with 11m were selected for the analysis. The mismatch in length was considered to satisfy the basic assumption of SEA that the wave incident should not be correlated on the either side of the junction. The peak energy transmitted predicted by TSEA was more appropriate for the modal overlap value of lesser than unity. On contrast the wave propagation was more appropriate for the modal overlap value of greater than unity.

Pinnington [61] conducted numerical parameter study on two degree of freedom system subjected to impulse excitation. Initial energy transfer ratio, the ratio of raise time to peak value was computed. Mathematical expressions were also derived for exact wave solution and transient statistical energy analysis on a two oscillator system. The results were computed by both transient statistical energy analysis and two degree of freedom system . It was notice that transient statistical energy analysis had faster initial response and reached the peak value in half the time compared to the two degree of freedom system. It was also observed that the transient statistical energy analysis reached the peak energy level 1.36 times faster than two degree of freedom system and post peak energy level in both the systems were in good agreement.

Mao [62] proposed a new technique to determine the impact load in engineering structures based on transient statistical energy analysis. Average kinetic energy responses of the subsystems were used in energy balance equation with a two-stage identification scheme, to determine the location and input energy of impact load. The values of determined input energy of impact load were used in Parseval theorem assuming constant value within each analysis frequency band to derive impact load amplitude spectrum. A two-plate and a three-plate coupling structural system were used to determine impact load amplitude spectrum using transient statistical energy analysis.

It has been observed that most of the researchers starting with Lyon in 1975 who initiated Statistical Energy Analysis a method to study the structural behaviour in high frequency vibration range, have been involved in determining the SEA parameters accurately. Initially classical wave approach for analytical solutions, power injection method for experimental solutions were used in determining SEA parameters. Later finite element method for numerical technique was introduced with the development of numerical tools. Many alternate methods was also proposed by many authors as an alternate to wave approach, such as Wave Intensity Analysis, use of Receptance theory and Green functions, Monte Carlo technique, fuzzy structure theory and so on. Also the existing methods were modified for special cases such as modified SEA for non-conservative couplings, discrete singular convolution and mode superposition approach for mid-frequency range, spectral finite element method for composites, Statistical modal Energy distribution Analysis, power transfer coefficient for the systems with low modal overlap. Though most of the structural high frequency vibrations are carried for beam and plate like structures with different configurations and joints, the material is restricted to be isotropic. Only few works have been extended to orthotropic materials like laminates where few plies with different orientations were stacked and their parametric studies have been documented. The same can be extended to sandwich structures such as honey comb or any such configurations to study the material behaviour when excited at high frequencies. The research can also be extended to study not only the SEA parameters of isotropic materials connected with different joints but also for crack detection in the material and to know the strength of the joints. The major limitation in ESEA lies in the accurate measurement of power input and has to be overcome by developing alternate techniques and use of hybrid methods. Though extensive studies have been made on SEA since few decades, there are many grey areas in the existing methods requring improvement and also to develop altogether new techniques to study statistical energy analysis.

, appear before the acknowledgment. In the event multiple appendices are required, they will be labeled “Appendix A,” “Appendix B, “ etc. If an article does not meet submission length requirements, authors are strongly encouraged to make their appendices supplemental material.

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